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The linear stability in systems with intensive mass transfer—IV. Gas–liquid film flow

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Abstract—A linear analysis of the stability of the flow in a laminar boundary layer under conditions of intensive interphase mass transfer between gas and liquid film, when high mass fluxes through the phase boundary induce secondary flows, is suggested. The influence of the interface velocity on the hydrodynamic stability of the gas flow is significant in the present case. The critical Reynolds numbers are obtained at different intensities of non-linear mass transfer in a laminar boundary layer. The influence of the direction of the intensive interphase mass transfer on the hydrodynamic stability is shown as well. The motion of the interface leads to a decrease of velocity gradients, which is the cause for an increase in stability of the flow. The flow is stable at large Reynolds number in the liquid phase. The dependencies of the critical Reynolds numbers from the normal and tangential component of the interface velocity are shown. Stability of the flow is more 'sensitive' to a change in the normal component. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The first three reports [1–3] show that the hydrodynamic stability of flow in the boundary layers over a flat interface depends on the direction and rate of the intensive interphase mass transfer. The results obtained for gas–liquid and liquid–liquid systems show [2, 3] that the stability of the flow in the boundary layer depends considerably on the interface velocity. This velocity is a result of superposed influence of the flux of momentum (hydrodynamic interaction between two phases) and the mass flux (inducing of parallel secondary flows) through the phase boundary. Based on the above, the study of the influence of the normal and the tangential components of the interface velocity on the hydrodynamic stability of the velocity profiles has to be developed. For the practical and interesting gas–liquid film flow system this compound crossed effect can be observed in detail.

2. NON-LINEAR MASS TRANSFER IN GAS–LIQUID FILM FLOW SYSTEMS

Non-linear mass transfer under conditions of intensive interphase mass transfer between gas and liquid film flow was the subject of numerical and asymptotic analysis [4, 5]. The results obtained show that the above mentioned crossed influence manifests itself when the diffusion resistance is localized in the gas phase. In this case the velocity distribution in gas can be expressed using the similarity variables

$$\tilde{u} = \tilde{u}_0 f(\xi), \quad \tilde{v} = 0.5\left(\frac{\tilde{u}_0 \tilde{\nu}}{x}\right)^{0.5}(\xi f' - f),$$

Table 1. Initial values of f, its derivatives and parameter k (in gas phase, $\varepsilon = 1$, $\theta_1 = 0.15$, $\theta_3 = \theta$)

θ	$f(0)$	$f'(0)$	$f''(0)$	k
−0.3	0.30411	0.225	0.39250	0.702
−0.2	0.18788	0.225	0.36000	0.871
−0.1	0.08729	0.225	0.33240	1.023
0	0	0.225	0.30890	1.158
0.1	−0.07632	0.225	0.28900	1.278
0.2	−0.14399	0.225	0.27175	1.387
0.3	−0.20536	0.225	0.25650	1.486

$$\xi = y\left(\frac{\tilde{u}_0}{\tilde{\nu} x}\right)^{0.5}, \qquad (1)$$

where $f(\xi)$ is the solution of the problem

$$2f''' - ff'' = 0,$$
$$f(0) = a, \quad f'(0) = b, \quad f''(0) = c. \qquad (2)$$

The associated set of initial conditions is obtained in refs. [4, 5] and presented in Table 1, where the parameter k is determined by expression

$$k = \lim_{\xi \to \infty}(\xi f' - f). \qquad (3)$$

3. STABILITY ANALYSIS

The linear analysis of the stability of velocity profiles $f'(0)$ was carried out in the cases [4] where the hydrodynamic interaction between gas and liquid film flow is determined by parameters

NOMENCLATURE

a	initial value of the Blasius function
A	dimensionless wave number
b	initial value of first derivative of the Blasius function
c	concentration, initial value of second derivative of the Blasius function
C	dimensionless phase velocity
D	diffusion coefficient
f	Blasius function
k	parameter
M	molecular mass
Re	Reynolds number
u	velocity of basic stationary flow in x direction
v	velocity of basic stationary flow in y direction
x	coordinate
y	coordinate.

Greek symbols

ε	parameter
θ	parameter
ν	kinematic viscosity
ξ	variable
ρ	density
χ	Henry number.

Subscripts and superscripts

*	conditions of transferred substance
0	conditions in volume
cr	critical number
i	imaginary part of complex number
max	maximum
min	minimum
r	real part of complex number.

$$\varepsilon = (\tilde{\nu}/\tilde{D})^{1/2} = 1, \quad \theta_1 = \frac{u_0}{\tilde{u}_0} = 0.15,$$

$$\theta_3 = -\theta\varepsilon = \frac{M}{\tilde{\rho}_0^*}(\tilde{c}_0 - \chi c_0), \qquad (4)$$

where M is the molecular mass of the transferred substance between phases, c_0 is the initial concentration of the transferred substance in the liquid, χ is the Henry's number, while the parameters for the gas phase $\tilde{\nu}$, $\tilde{D}$, $\tilde{u}_0$, $\tilde{c}_0$, $\tilde{\rho}_0^*$ are the dynamic viscosity coefficient, the diffusivity of the transferred substance, the initial velocity, the initial concentration and the phase boundary density, respectively.

The initial conditions associated with equation (2) were obtained considering equation (4) for different values of θ (Table 1) and they permit us to solve the problem (2) as a problem of Cauchy, as well as the introduction of $f(\xi)$ into the Orr–Sommerfeld equation. The Orr–Sommerfeld equation was solved analogously to that one in ref. [1]. The neutral curves and critical Reynolds numbers for different values of the dimensionless phase velocity (C_r) and length number (A) are obtained.

4. RESULTS AND DISCUSSION

The neutral curves (Re, A) and (Re, C_r) are shown in Figs. 1 and 2. They allow us to obtain the critical Reynolds numbers (Re_{cr}), which are presented in Table 2. Analogously to refs. [1–3], these results show the fact that the stability of the flow decreases with the rise of the parameter θ.

The results presented in Table 2 and the analogous ones in refs. [1–3] permit us to study the influence of the tangential interface velocity component ($f'(0)$) on the stability of the flow (Re_{cr}). These results are shown in Fig. 3 at θ = const. They clearly present a well-pronounced dependence, i.e. the rise of interface velocity leads to stabilizing of the flow.

Table 2. Values of the critical Reynolds numbers Re_{cr}, wave velocities C_r, wave numbers A and $C_{r\,min}$, A_{min} obtained (in gas phase, at the conditions of $\varepsilon = 1$, $\theta_1 = 0.15$, $\theta_3 = \theta$)

θ	Re_{cr}	A	C_r	$A_{r\,min}$	$C_{r\,min}$
−0.3	3760	0.280	0.4566	0.310	0.4576
−0.2	2484	0.290	0.4748	0.328	0.4760
−0.1	1714	0.305	0.4922	0.347	0.4928
0	1239	0.310	0.5064	0.361	0.5083
0.1	941	0.320	0.5200	0.376	0.5219
0.2	743	0.325	0.5311	0.390	0.5338
0.3	605	0.340	0.5429	0.402	0.5449

The significant influence of the normal velocity on the interface $f(0)$ upon the stability of the flow was shown in refs. [1–3]. Considering this, the dependence of Re_{cr} from $f'(0)$ at $f(0)$ = const is of great interest. It is presented in Fig. 4 and demonstrates one much better expressed dependence.

The results obtained so far allow to obtain the dependence of the critical Reynolds number Re_{cr} from the normal component of velocity on interface $f(0)$ at the condition of constant value of tangential interface velocity component $f'(0)$. These results are shown in Fig. 5 and they express one continuous rise of the stability of the flow in the transition from injection to suction in the laminar boundary layer.

5. CONCLUSIONS

The results obtained in these four reports give us an opportunity to make some basic conclusions:

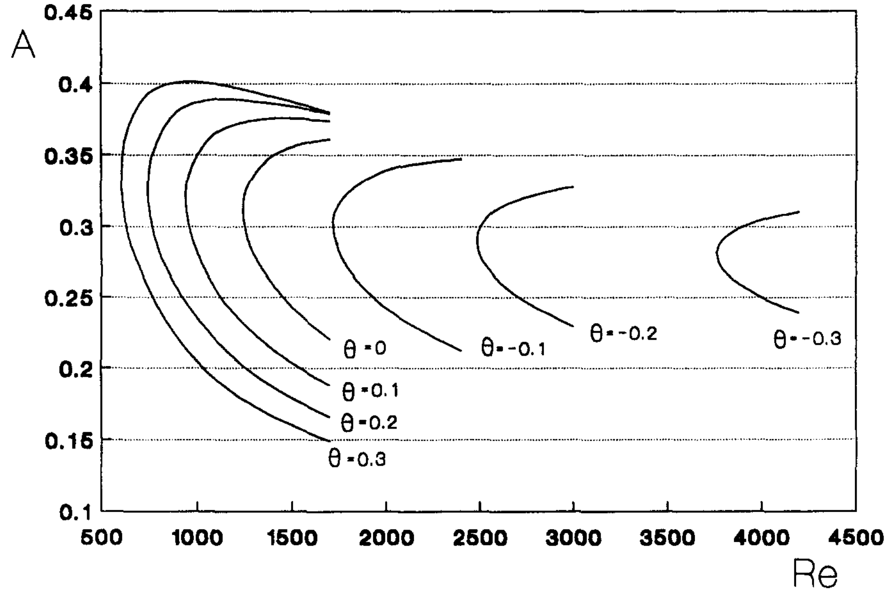

Fig. 1. The neutral curves for the wave number A as a function of Reynolds number Re in the gas phase ($\varepsilon = 1$).

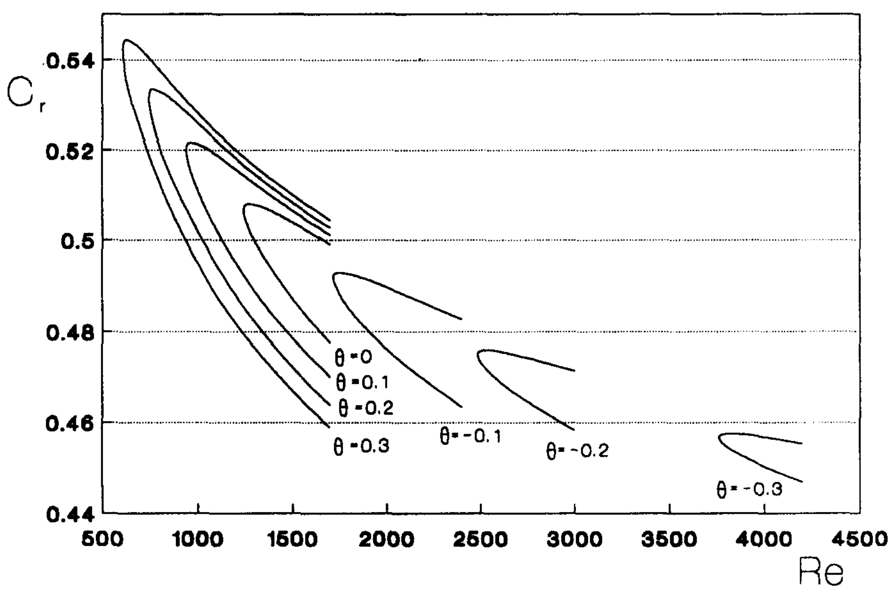

Fig. 2. The neutral curves for the phase velocity C_r as a function of Reynolds number Re in the gas phase ($\varepsilon = 1$).

(1) The flows in the boundary layers with increasing velocity in the depth of the fluid ('Blasius flow') are characterized with the hydrodynamic stability, which increases with the rise of the tangential velocity component on the interface and decreases from its normal component in the transition from 'suction' ($v < 0, f(0) > 0$) to 'injection' ($v > 0, f(0) < 0$) in the laminar boundary layer. Hydrodynamic stability of this type of flow in the boundary layer depends 'independently', as on the normal component of velocity, on the interface and on the tangential component of the interface velocity.

(2) The flows in the boundary layers with decreasing velocity in the depth of the fluid ('Couette flow')

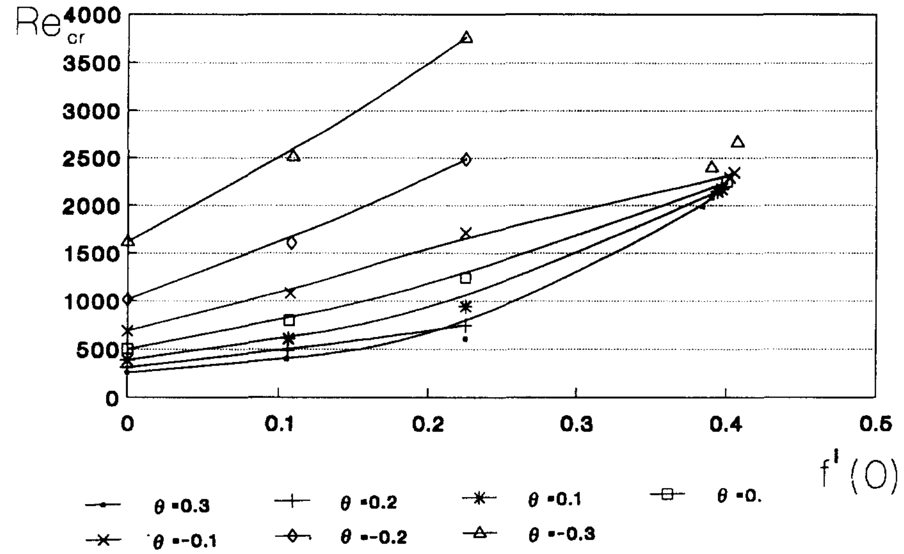

Fig. 3. The influence of the tangential interface velocity component ($f'(0)$) on the stability of the flow (Re_{cr}).

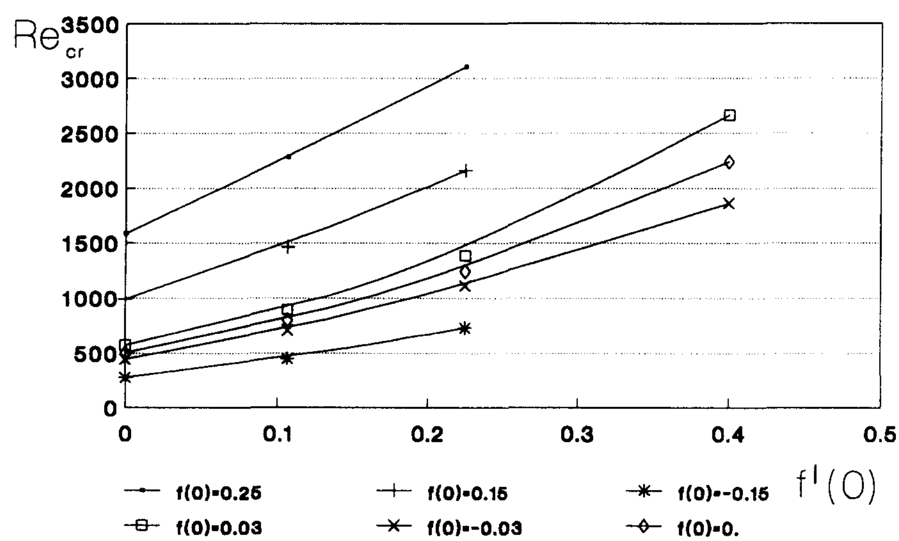

Fig. 4. The dependence of Re_{cr} from $f'(0)$ at $f(0)$ = const.

are practically global stable, and they are not affected by changes in the normal and tangential component of the velocity on the interface (in the range of changes they were studied in refs. [2, 3]).

(3) The systems with intensive interphase mass transfer are characterized by the fact that the kinetics of mass transfer do not follow from the linear theory of the mass transfer, and the obvious changes in the hydrodynamic stability are observed. These effects have been explained very often [6–9] with the Marangoni effect, i.e. the induction of tangential secondary flow on the phase boundary. The investigations of the kinetics of mass transfer in the systems with intensive interphase mass transfer [10] and their hydrodynamic stability [2, 3] show that the same effects can be explained by the effect of the non-linear mass transfer, i.e. the induction of normal secondary flows on the phase boundary. Consequently, it is poss-

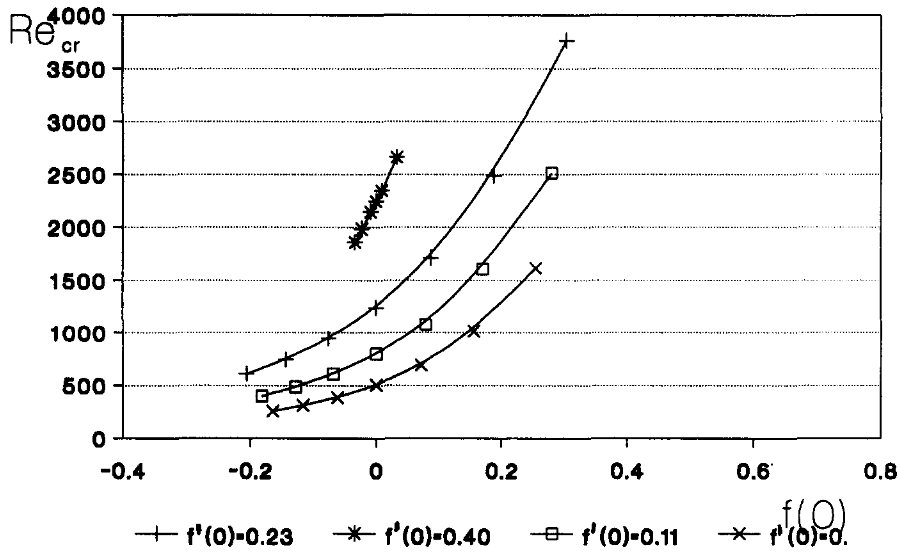

Fig. 5. The dependence of the critical Reynolds number Re_{cr} from the normal component of velocity on interface $f(0)$ at the condition of constant value of tangential interface velocity component $f'(0)$.

ible to compare the Marangoni effect with the effect of the non-linear mass transfer.

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